PERMUTATION

AND

COMBINATION

Factorial Representation

Continued product of natural number.

$2!$ or $\lfloor 2 = 2\times1 = 2$

$3!$ or $\lfloor 3 = 3\times2\times1 = 6$

$4!$ or $\lfloor 4 = 4\times3\times2\times1 = 24$

$5!$ or $\lfloor 5 = 5\times4\times3\times2\times1 = 120$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$n!$ or $\lfloor n = 1\times2\times3\times4\ldots\ldots\times n \quad , \; n\in N$

$0! = 1 \longrightarrow$ By default

$\underbrace{(-1)!\;,\;(-2)!\;,\;(-3)!\;,\;\ldots\ldots}_{\text{Not defined}}$

Eg:— 1. $\dfrac{10!}{6!\,4!} = \dfrac{10\times\overset{3}{\cancel{9}}\times\cancel{8}\times7\times\cancel{6\times5\times4\times3\times2\times1}}{\cancel{(6\times5\times4\times3\times2\times1)}\times(\cancel{4}\times\cancel{3}\times\cancel{2}\times1)} = 10\times7\times3 = 210$

2. $\dfrac{1}{9!} + \dfrac{1}{10!} = \dfrac{x}{11!}$ find x.

$\dfrac{1}{9!}\left(1+\dfrac{1}{10}\right) = \dfrac{11}{11\times10\times9!}(10+1) = \dfrac{11\times11}{11!} = \dfrac{121}{11!} = \dfrac{x}{11!}$

$\therefore\; x = 121$

3. $(n+2)! = 2550\times n!$

$(n+2)(n+1)\cancel{n!} = 2550\times\cancel{n!}$

$50\times51 = (n+2)(n+1)$

$\therefore\; n = 49$

Date :

4. $(n+1)! = 12 \times (n-1)!$

$(n+1)\, n\, \cancel{(n-1)!} = 12 \times \cancel{(n-1)!}$

$n(n+1) = 3 \times 4.$

$\therefore n = 3.$

5. Prove that

$$\frac{2n!}{n!} = 2^n \{1 \cdot 3 \cdot 5 \cdots\cdots (2n-1)\}$$

$\rightsquigarrow$ $$LHS = \frac{2n\,(2n-1)\,(2n-2)\cdots\cdots}{n\,(n-1)\,(n-2)\cdots\cdots}$$

$$= \frac{2\cancel{n}\,(2n-1)\cdot 2\cancel{(n-1)}\cdot(2n-3)\cdot 2\cancel{(n-2)}\cdots\cdots}{\cancel{n}\,\cancel{(n-1)}\,\cancel{(n-2)}\cdots\cdots}$$

$$= 2^n \{(2n-1)\cdot(2n-3)\cdots\cdots 5\cdot 3\cdot 1\}$$

$$= 2^n \cdot (1\cdot 3\cdot 5 \cdots\cdots (2n-3)\cdot(2n-1)\}.$$

Hence proved.

Ques. Prove that

$n!+1$ is not divisible by any natural no. b/w 2 & n

Sol^n – Attempted by me. (M-1)

$n!+1 = 2q_1 + r_1$

$n!+1 = 3q_2 + r_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$n!+1 = nq_n + r_n.$

We know that

$n! \equiv 0 \mid Mod\,(n) \qquad n \in N.$

$n!+1 \nmid n \qquad$ When $n \neq 1,\ n \in N$

$\therefore\ n!+1 = nq_n + r_n$

On comparing, we get

$nq_n \neq 1 \rightarrow$ So, $nq_n = n!$

then, $r_n = 1$

(M-2)

$n!+1$

Let $q \in (2, n) \quad , q \in N.$

$$R\left(\frac{n!+1}{q}\right) = R\left(\frac{n!}{q} + \frac{1}{q}\right)$$

Since, $R\left(\frac{n!}{q}\right) = 0$ $\quad \{ q \in (2, n) \;, q \in N$

$$\therefore R\left(\frac{n!+1}{q}\right) = R\left(0 + \frac{1}{q}\right) = R\left(\frac{1}{q}\right) = 1$$

Ques. Prove that

$$(n!)^2 \leq n^n (n!) < (2n)!$$
$$\forall \; n \in I^+$$

Sol^n – Case 1.

$$(n!)^2 \leq n^n (n!)$$

$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) n.$

$1 \leq n$
$2 \leq n$
$3 \leq n$
$4 \leq n$
$\vdots \quad \vdots$
$(n-1) \leq n$
$(n) \leq n.$

$\times$ _______________

$1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1)\, n \leq n^n \quad \Rightarrow \quad n! \leq n^n$

Multiplying $n!$ both side.

$$(n!)^2 \leq n! \cdot n^n \longrightarrow (1)$$

Case 2.

$$n^n (n!) \leq (2n)!$$

$(2n)! = 2n\,(2n-1) \cdot (2n-2) \cdots (n+2)\,(n+1) \cdot n\,(n-1) \cdots$
$\cdots (n-3) \cdots 2 \cdot 1$

$(2n)! = 2n\,(2n-1) \cdot (2n-2) \cdots (n+2)\,(n+1)\; n!$

$n < n+1$
$n < n+2$
$\vdots \quad \vdots$
$n < 2n-1$
$n < 2n$

$\times$ _______________

$n^n \leq 2n\,(2n-1)\,(2n-2) \cdots (n+1)$

$$n^n\, n! \leq (2n)! \longrightarrow (11)$$

from (I) & (II), we get

$$(n!)^2 \le n^n (n!) \le (2n)!.$$

Ques Prove that

1) $33!$ is divisible by 2^{15}.

Sol^n – $\Rightarrow 33(32)! = 33!$

$= 33 \cdot (2\times n)!$ Let $n = 16$.

$= 33 \cdot 2^n\, n! \,(1 \cdot 2 \cdot 3 \cdots (2n-1))$

put $n=16 \Rightarrow \underbrace{33 \cdot 2^{16} \cdot 16!\,(1 \cdot 2 \cdot 3 \cdots 31)}_{\text{divisible by } 2^{15}}$

2) find the largest integer n such that $33!$ is divisible 2^n.

Sol^n – Method 1.

$$33! = 2^{16}(33 \cdot 31 \cdot 29 \cdots 3 \cdot 1)(16 \cdot 15 \cdots 3 \cdot 2 \cdot 1)$$

$$= 2^{16+15} \cdot (33 \cdot 31 \cdot 29 \cdots 3 \cdot 1)(15 \cdot 7 \cdot 13 \cdot 3 \cdot 11 \cdot 5 \cdot 9 \cdot 7 \cdot 3 \cdot 5 \cdot 3)$$

$\therefore n = 31$.

Concept :–

Highest power of any prime (p) in $n!$

$$= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$$

where $[\cdot]$ = G.I.F.

Ex :–

Power of 2 in $3!$ $= \left[\frac{3}{2}\right] + \left[\frac{3}{2^2}\right] + \cdots$

$= 1 + 0 = 1$

Method 2.

Power of 2 in $(33)!$ $= \left[\frac{33}{2}\right] + \left[\frac{33}{2^2}\right] + \left[\frac{33}{2^3}\right] + \left[\frac{33}{2^4}\right] + \left[\frac{33}{2^5}\right] + \left[\frac{33}{2^6}\right]$

$= 16 + 8 + 4 + 2 + 1 + 0 + 0 = 31$

Fundamental Concept of Counting

Counting principle

"Counting without counting"

1. Addition principle

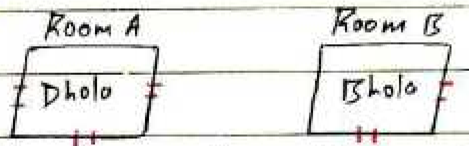
Keyword :— 'or'.

If job A & job B can be operated by m & n ways respectively, then no. of ways of operating either A or B is $m+n$.

$\boxed{\text{job A}} \Downarrow m$ $\quad$ $\boxed{\text{job B}} \Downarrow n$

Job A or Job B.
No. of ways $= m+n$.

Ex :—



Find the no. of ways for Dhola or Bhola get out of the room.

Solⁿ :— No. of ways to get out of Room
for Dhola = 3.
for Bhola = 2

∴ No. of ways to get out of the room either for Dhola or Bhola = 5

2. Multiplication principle

Keyword :— 'and'

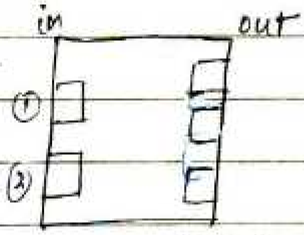
If there are two jobs say A & B, A can be completed by 'm' ways & B can be completed by 'n' ways then both job A & B can be completed in succession (one after another) by $m \times n$ ways.

Qus. There are three candidate for a classical and 5 for mathematical and 4 for a natural science scholarship
1) In how many ways can these scholarships be awarded?
11) How many ways one of these scholarships awarded?

Solⁿ— 1) No. of ways all the scholarships be awarded
$= 5 \times 4 \times 3 = 12 \times 5 = 60$

2) **Now of** ways one of these scholarships be awarded $= 3+5+4 = 12.$

Ex:—



भूल भुलैया घर

1) No. of ways for ramu to get out of maze
$= 2 \times 3 = 6.$

2) for Gate ①
No. of exit = 3
Or, for Gate ②
No. of exit = 3.

$\therefore$ Total no. of exit $= 3+3 = 6.$

NOTE :—

- 'and' or 'intersection' $\Longrightarrow \times$
  (and wali feeling)
- 'or' or 'Union' $\Longrightarrow +$
  (or wali feeling)

Qus. A room has 6 doors. Shreya enter the room through one door and come out through a different door. find the no. of ways.

Solⁿ— No. of ways to enter the room = 6
No. of ways to get out of the room = 5

Using Multiplication principle,
No. of ways $= 6 \times 5 = 30$.

Ques. The flag of a newly form forum is the form of □ □ □ of three blocks. Each to be coloured differently if there are six different colour on the whole to choose from. How many such designs are possible.

Sol^n — B-1 B-2 B-3

$\downarrow \quad \downarrow \quad \downarrow$

Job. A B C

Job A, B & C must be preformed in succession

Then, No. of ways $= 6 \times 5 \times 4 = 120$.

Ques. In how many ways, 5 ring of different types can be worn in 4 figures.

Sol^n —

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$4 \quad 4 \quad 4 \quad 4 \quad 4$

No. of ways $= 4^5$.

Permutation

⇓

Arrangement

Ex: — A, B, C

Arrange any two out of these.

$$\left[\begin{matrix} AB & BA \\ AC & CA \\ BC & CB \end{matrix}\right] \rightarrow 6 \text{ ways}$$

Permutation

↙ ↘

Selection Arrangement

Ex :- Write the no. of permutation of vowels A, E, I, O, U taking any three at a time starting with A.

AEI , AEO , AEU , AIU , AIO , AOU

AIE , AOE , AUE , AUI , AOI , AUO.

12 permutation or arrangement

Permutation = Selection + Arrangement.

Ex :- Arrange any three out of 4 objects (diff. obj.)

or, Arrange 4 students on 3 seats

$\rightsquigarrow$ No. of Arrangement = Seat 1 Seat 2 Seat 3

$$\boxed{4} \times \boxed{3} \times \boxed{2}$$

$$= 4 \times 3 \times 2 = 24.$$

Ex :- Arrange any 2 out 5 diff. objects.

$\rightsquigarrow$ No. of Arrangement $= 5 \times 4 = 20$

$$= {}^5P_2 = \frac{5!}{3!}$$

Symbol :- $P(5, 2)$

Concept :-

No. of permutation of n different objects when r is taken at a time or No. of permutation (arrangement) of r object out of n different object

$$= {}^nP_r = P(n, r)$$

where, $0 \le r \le n$, $n, r \in N$

$$ {}^nP_r = \frac{n!}{(n-r)!}$$

NOTE :—

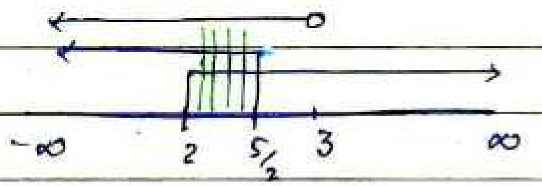
- $^nP_r \in J^+$
- $^nP_r = \dfrac{n!}{(n-r)!}$
- $0 \le r \le n$
- $0 < n \;,\; n \in N$
- $0 \le n-r \;,\; n-r \in N$
- $0 \le r \;,\; r \in W$

Ex:— find Domain & Range.

$$f(x) = {}^{3-x}P_{x-2}$$

Sol^n— $n = 3-x \;;\; r = x-2$

$n > 0$	$r \geq 0$	$n-r \geq 0$
$3-x > 0$	$x-2 \geq 0$	$(3-x)-(x-2) \geq 0$
$3 > x$	$x \geq 2$	$5-2x \geq 0$
		$5/2 \geq x$



$x \in [2, 5/2] \;,\; x \in J$

$\Rightarrow x = 2$

Domain $= \{2\}$

Range $= \{f(2)\}$

$f(2) = {}^{3-2}P_{2-2} = {}^1P_0 = \dfrac{1!}{(1-0)!} = 1.$

Ex:— find Domain & Range

$$F(x) = {}^{(x^2-1)}P_{(3-2x)}$$

Sol^n— $n = x^2-1 > 0 \Rightarrow (x+1)(x-1) > 0$

$x \in (-\infty, -1] \cup [1, \infty)$

$r = 3-2x \geq 0 \Rightarrow 3/2 \geq x.$

$n - r \geq 0$

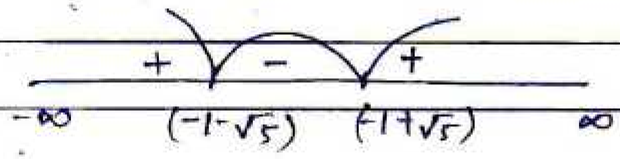
$x^2 - 1 - (3 - 2x) \geq 0$

$x^2 + 2x - 4 \geq 0$

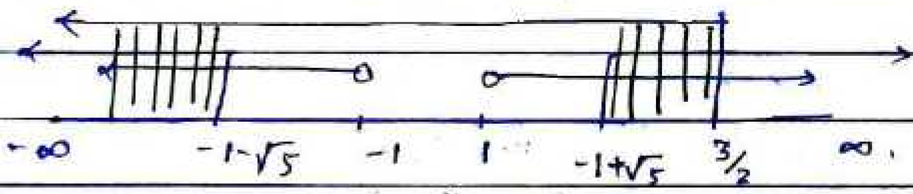
$x^2 + 2x + 1 - 5 \geq 0$

$(x+1)^2 - (\sqrt{5})^2 \geq 0$

$(x + 1 - \sqrt{5})\,(x + 1 + \sqrt{5}) \geq 0$



$x \in (-\infty, -1-\sqrt{5}] \cup [-1+\sqrt{5}, \infty).$



$x \in (-\infty, -1-\sqrt{5}] \cup [-1+\sqrt{5}, \tfrac{3}{2}]$

Since, $x \in I$

$x \in \{\ldots\ldots, -5, -4\}$

Domain

Range $= \{\ldots\ldots, f(-5), f(-4)\}$.

NOTE :—

$\rightsquigarrow$ ${}^nP_0 = \dfrac{n!}{(n-0)!} = 1 =$ No. of permutation when arrangement of nothing out of n - different object.

$\rightsquigarrow$ ${}^nP_1 = \dfrac{n!}{(n-1)!} = n =$ Arrangement of 1 out of n - different object.

$\rightsquigarrow$ ${}^nP_n = \dfrac{n!}{(n-n)!} = n! =$ Arrangement of n out of n - different object.

Ex:— 1. 'PRAT'

find No. of words (meaning or without meaning) using all letters.

Sol^n → No. of words $= 4! = {}^4P_4 = 24$.

2. 'PRATAP'.

No. of words $= \dfrac{6!}{2!\,2!} = \dfrac{6 \times 5 \times \cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2!}}$

$= 30 \times 6 = 180$.

Que. $2P(5,3) = P(n,4)$ find n.

Sol^n → $2\,{}^5P_3 = {}^nP_4$

$\cancel{2} \times \dfrac{5!}{\cancel{2!}} = \dfrac{n!}{(n-4)!}$

$\Rightarrow 5! = \dfrac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}}$

$\Rightarrow 5 \times 4 \times 3 \times 2 = n(n-1)(n-2)(n-3)$

$\therefore n = 5$

→ ${}^nP_r = n \times {}^{n-1}P_{r-1}$.

Que. $P(5,r) = 2 \cdot P(6,r-1)$ find r.

Sol^n — $\dfrac{\cancel{5!}}{(5-r)!} = 2 \cdot \dfrac{\cancel{6!}\ 6}{(6-r+1)!}$

$\Rightarrow (7-r)! = 12 \cdot (5-r)!$

$\Rightarrow (7-r)(6-r) = 12$

$\therefore r = 3$

→ Prove the following:—

1) ${}^nP_r = n \cdot {}^{(n-1)}P_{(r-1)}$

$\longrightarrow$ $LHS = {}^nP_r = \dfrac{n!}{(n-r)!} = n.\dfrac{(n-1)!}{(n-1-r+1)!}$

$$= n.\frac{(n-1)!}{((n-1)-(r-1))!} = n.\,{}^{n-1}P_{r-1} = RHS.$$

2) ${}^nP_n = {}^nP_{n-1}$.

$\longrightarrow$ $LHS = {}^nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n! \times 1}{0! \times 1} = \dfrac{n!}{(n-(n-1))!} = {}^nP_{n-1} = RHS$

3) ${}^nP_n = 2.\,{}^nP_{n-2}$.

$\longrightarrow$ $LHS = {}^nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n! \times 2!}{0!\,(n-(n-2))!}$

$$= 2.\frac{n!}{(n-(n-2))!} = 2.\,{}^nP_{n-2} = RHS.$$

4) ${}^nP_r = r\,{}^{n-1}P_{r-1} + {}^{n-1}P_r$.

$\longrightarrow$ $RHS = r\,\dfrac{(n-1)!}{(n-1-r+1)!} + \dfrac{(n-1)!}{(n-1-r)!}$

$$= r.\frac{(n-1)!}{(n-r)!} + \frac{(n-1)!}{(n-r-1)!} = \frac{(n-1)!}{(n-r-1)!}\left(\frac{r}{n-r}+1\right)$$

$$= \frac{n.(n-1)!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r)!} = {}^nP_r = LHS$$

$\rightsquigarrow$ $(n-r)\,{}^{n-1}P_{r-1} = {}^{n-1}P_r$.

Ques. find the no. of word which can be made by using the letters of the word 'MOBILE' when consonants always occupy odd places.

Sol^n —

$$\begin{array}{cccccc} \underline{C} & \underline{V} & \underline{C} & \underline{V} & \underline{C} & \underline{V} \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

Vowel — O, I, E

Consonants — M, B, L

$\therefore$ No. of ways $= \underset{^3P_3}{6} \times \underset{^3P_3}{6} = 36.$

Ques. How many words with or without meaning can be formed using the letter of the word "TUESDAY", if.

1) 4 letters used at a time
2) 5 letters used at a time
3) If all letters used at a time.
4) If all letters used at a time but the 1st letter is a vowel.
5) If all letters used at a time but the middle letter a word is always a consonants.
6) If we form a dictionary of the word formed by these letters. Then find the rank / position of the word.

Soln — 1) _ _ _ _

No. of ways $= {^7P_4} = \frac{7!}{3!} = 7\times6\times5\times4 = 20\times42 = 840$

2) _ _ _ _ _

No. of ways $= {^7P_5} = 7\times6\times5\times4\times3 = 840\times3 = 2520$

3) _ _ _ _ _ _ _

No. of ways $= {^7P_7} = 7!$

4) _ _ _ _ _ _ _

A, E, U

No. of ways $= 3\times{^6P_6} = 3\times6! = 3\times720 = 2160$

5) _ _ _ _ _ _ _

T, S, D, Y

No. of ways $= 4\times{^6P_6} = 4\times6! = 4\times720 = 2880$

Rank of a word in dictionary

6) ADESTUY

A — — — — — — $6!$

D — — — — — — $6!$

E — — — — — — $6!$

S — — — — — — $6!$

TA — — — — — $5!$

TD — — — — — $5!$

TE — — — — — $5!$

TS — — — — — $5!$

TUA — — — — $4!$

TUD — — — — $4!$

TUEA — — — $3!$

TUED — — — $3!$

TUESA — — $2!$

TUESDAY $1!$

Rank :– $4 \times 6! + 4 \times 5! + 2 \times 4! + 2 \times 3! + 2! + 1!$

$= 4 \times 720 + 4 \times 120 + 2 \times 24 + 2 \times 6 + 2 + 1$

$= 2880 + 480 + 48 + 12 + 2 + 1$

$= 15 + 528 + 2880$

$= 543 + 2880 = 3423$

Ques. find the Rank of the word "ZENITH".

Sol– EHINTZ

E — — — — — $5!$

H — — — — — $5!$

I — — — — — $5!$

N — — — — — $5!$

T _ _ _ _ _ $5!$

Z E H _ _ _ $3!$

Z E I _ _ _ $3!$

Z E N H _ _ $2!$

Z E N I H _ $1!$

Z E N I T H $1!$

Rank :— $5 \times 5! + 2 \times 3! + 2! + 1 + 1$

$= 5 \times 120 + 2 \times 6 + 2 + 1 + 1$

$= 600 + 12 + 4 = 616.$

Ques. A set contain 20 diffrent Element. find the total no. of triplet (x, y, z) can be formed such that atleast two element of the triplet are equal.

Sol^n – Method 1.

Case1.

$20 \times 19 = 380$

$19 \times 20 = 380$

$19 \times 20 = 380.$

No. of ways $= 3 \times 380$
$= 1140$

Case 2.

No. of ways $= 20$

$\therefore$ Total no. of ways $= 1140 + 20 = 1160.$

Method 2.

No. of triplet where Atleast two element of the triplet are equal $=$ Total no. of triplet $-$ No. of triplet of diff. element

$= 20^3 - 20 \times 19 \times 18$

$= 20(400 - 342) = 1160.$

Ques. find 4- digit no. that can be formed by using the digits 2, 4, 6, 8. If repitition is digits is not allowed. Also find the sum of the formed no.

Sol-

$\underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}$

No. of ways = 24.

The sum of the formed no. $= 1000 \times 120 + 100 \times 120 + 10 \times 120 + 1 \times 120$

$= 120(1111)$

$= 133320.$

Ques. find the sum of all no.s of 5-digit (0 to 9). If repitition is not allowed.

Soln-

$\underset{9}{\_}\ \underset{9}{\_}\ \underset{8}{\_}\ \underset{7}{\_}\ \underset{6}{\_}$

No. of ways $= 42 \times 8 \times 9 \times 9 = 81 \times 8 \times 42$

$= 648 \times 42 = 27216$

$0+1+2+3+4+5+6+7+8+9$

$= 45$

Sum of 10^4 place $= 3024 \times 45$

$= 136080.$

Sum of $10^3, 10^2, 10^1, 10^0$, th place

— — — — Grouping $\langle 1, 2, 3 \ldots, 9 \rangle$

$\Rightarrow 8 \times 8 \times 7 \times 6$ times $= 2688.$

$\therefore$ Sum of the formed no. $= 136080 \times 10^4 + 45 \times 2688 \times (10^3 + 10^2 + 10^1 + 1)$

$= 1360800000 + 134386560$

$= 149518650$

Ques. How many four digit number can be made with distinct digit. Sum of the all number formed.

Sol^n:-

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }$

9 9 8 7

No. of number formed $= 9 \times 9 \times 8 \times 7$

$= 9 \times 504 = 4536.$

Or,

Total No. with distinct digits $= {}^{10}P_4 = 10 \times 9 \times 8 \times 7$

Total No. with distinct digits when 0 is at 1st place $= {}^{9}P_3 = \dfrac{9!}{6!} = 9 \times 8 \times 7$

$\therefore$ Total No. of 4 digit number with distinct digit $= 10 \times 9 \times 8 \times 7 - 9 \times 8 \times 7$

$= 9 \times 9 \times 8 \times 7 = 4536.$

Sum of Numbers $= 45 \times 504 \times 1111 - 45 \times 56 \times 111$

$= 24{,}917{,}760$

Ques. In how manys 7 picture can be hung from 5 nails on ~~the~~ a wall.

Sol^n:-

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }$

No. of ways $= {}^{7}P_5 = \dfrac{7!}{2!} = 7 \times 6 \times 5 \times 4 \times 3$

$= 42 \times 20 \times 3$

$= 60 \times 42 = 2520$

Ques. Determine the no. of natural number less than 10^4 in the decimal notation of which all the digit are distinct.

Sol^n – No. of Natural Number less than 10^4, (diff. digits)

$N < 10000$ (least 5 digit number)

4 - digit Numbers.

$= {}^{10}P_4 - {}^{9}P_3 = 9 \times 9 \times 8 \times 7 = 4536.$

3 - digit numbers

$= {}^{10}P_3 - {}^{9}P_2 = 9 \times 9 \times 8 = 81 \times 8 = 648$

2 - digit numbers

$= {}^{10}P_2 - {}^{9}P_1 = 9 \times 9 = 81.$

1 - digit numbers

$= 9$

Total no. of number formed $= 5274.$

Ques. In an examination hall, there are four rows of chairs, each row ~~hav~~ has 8 chairs one behind the other. There are two classes sitting for examination with 16 student in each class. It is desired that in each row all student belong to same class. and that ~~know~~ no two ~~to~~ adjacent rows are alloted to the same class. In how many ways can these 32 student be sitted?

Sol^n – Two types of classes C_1 & C_2.

Rows		Case 1.	Case 2.
	R_1	C_1	C_2
	R_2	C_2	C_1
	R_3	C_1	C_2
	R_4	C_2	C_1

No. of ways of seating of 16 student of $C_1 = 16!$

" " " " " " " " " $C_2 = 16!$

Total no. of ways of seating plan $= 16! \times 16!$ in case (1)

" " " " " " " $= 16! \times 16!$ in case (2)

$\therefore$ Total no. of ways $= 2 \times (16! \times 16!)$

Ques. 10 different letters of an Alphabet are given word with 5 letter are formed from these given letter. Determine the no. of word which have atleast one letter repeated.

Solⁿ — $\_\_ \ \_\_ \ \_\_ \ \_\_ \ \_\_$

Total no. of word formed $= 10^5$

No. of word formed with no letter repeat $= 10 \times 9 \times 8 \times 7 \times 6$

$\therefore$ No. of word formed with atleast one letter repeat $= 10^5 - 10 \times 9 \times 8 \times 7 \times 6$.

Permutation under Restriction

1. No. of all permutations of n different objects taken r at a time when a particular object is to be always included in each arrangement is $r \cdot {}^{(n-1)}P_{(r-1)}$

$n \longrightarrow$ different objects

$r \longrightarrow$ Taken at a time.

1 particular object is always included

Proof :—

$$\underline{\quad}\ \underline{\quad}\ \cdots\cdots\ \underline{\quad}$$
$$P_1 \quad P_2 \ \cdots\cdots\ P_r \qquad (r \text{ places})$$

We have to fill r places using n object, one particular object is always to be included.

for one particular object,
there are r choices for being placed for remaining (r-1) places, to be filled from (n-1) objects.

$$\therefore \text{ No. of ways} = \underbrace{{}^{n-1}P_{r-1} + {}^{n-1}P_{r-1} + \cdots\cdots + {}^{n-1}P_{r-1}}_{r \text{ times}}$$

$$= r \cdot {}^{(n-1)}P_{(r-1)}$$

2. → No. of permutation of 'n' different object taken 'r' at a time when one particular object is never included in the arrangement is ${}^{n-1}P_r$

3. → No. of permutation of 'n' different object taken 'r' at a time when two specified objects always occur together in the arrangement is $2 \times (r-1)!\ {}^{(n-2)}P_{(r-2)}$

__Ques.__ In how many ways can the letters of the word 'PENCIL' be arranged so that
1) N is always next to E
2) N and E are together

Sol^n – 1) PENCIL

$\downarrow$ lock (EN)

No. of ways $= {}^5P_5 = 5! = 120$.

2) No. of way $= 2 \times 5! = 240$.

PENCIL

PNECIL

Permutation of object when all are not different.

1. $\rightarrow$ No. of mutually distinguisable permutations of 'n' things taken all at a time of which 'p' are alike of one kind, 'q' are alike of other kind such that $p+q = n$. is $\dfrac{n!}{p!\,q!}$

Ques There are two works of each of 3 volumes and two works each of 2 volumes; In how many ways can the 10 books be placed on a shelf so that the volumes of the same work are not separated?

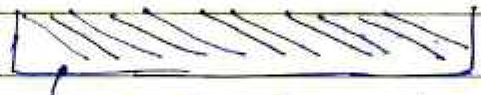
Sol^n –

$W_1 \rightarrow W_{11}, W_{12}, W_{13}$

$W_2 \rightarrow W_{21}, W_{22}, W_{23}$

$W_3 \rightarrow W_{31}, W_{32}$

$W_4 \rightarrow W_{41}, W_{42}$



10 Books

No. of ways $= 4! \times 3! \times 3! \times 2! \times 2!$

$= 24 \times 36 \times 4$

$= 3456$.

Ques. find the no. of ways where the letters of the word 'EAMCOT' is used and not all vowels are not together.

Sol:- Total No. of words = $6!$

No. of ways when all vowels come together = $4! \times 3!$

No. of such words where not all vowel are adjacent to each other $= 6! - 4! \times 3!$
$= 720 - 144$
$= 576$

Gap Method.

→ Jab Bhi question mai kisi Do chizo ko ek sath na aane dena ho toh GAP METHOD use Karenge !!

Ques. All letters of the word 'EAMCOT' are arranged in different possible ways. find the number of arrangement in which no two vowels are adjacent to each other.

Sol:- AEO ; MCT

— M — C — T —

→ 4 Gaps.

Required of places is 3 (A, E, O)

No. of arrangement of vowels in such a way that no two vowel are adjacent to each other = $^4P_3$ (Arrangement of 3 gap out of 4)

Date :

Arrangement of other letters $= 3!$

Total such words $= 4! \times 3! = 144$.

Que. In how many ways can a lawn tennis mixed double be made up from seven married couples if no husband and wife play in the same set?

Sol^n — Game (Two team)

Team A	Team B
1 men	1 men
1 women	1 women

seven couples

M_1	W_1
M_2	W_2
M_3	W_3
M_4	W_4
M_5	W_5
M_6	W_6
M_7	W_7

No. of arrangement of men $= {}^7P_2 = \dfrac{7!}{5!} = 42$

No. of arrangement of women $= {}^5P_2 = \dfrac{5!}{3!} = 20$

$\therefore$ No. of plan of lawn tennis mixed double games $= 42 \times 20 = 840$.

Que. m men and n women are to be seated in a row so that no two women sit together. If $m > n$ then show that the number of ways in which they can be seated as

$$\frac{m!\,(m+1)!}{(m-n+1)!}$$

Sol^n — $\underbrace{m}_{\text{Men}} + \underbrace{n}_{\text{Women}}$ $\qquad m > n$.

___ M_1 ___ M_2 ___ ... ___ M_m ___

$m \longrightarrow$ men

$m+1 \longrightarrow$ Gaps

No. of ways such that no women sit together $= m!\ {}^{m+1}P_n$

$$= \frac{m!\,(m+1)!}{(m+1-n)!}$$

Que: How many arrangement can be made by the letter of the word 'MATHEMATICS'. In how many of them vowels are together?

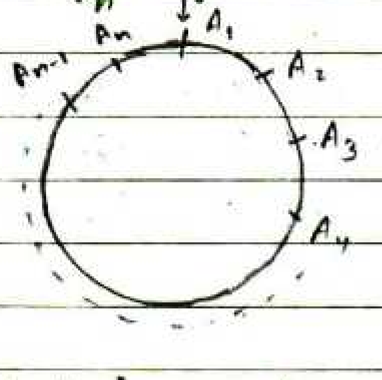
Solⁿ— 1) No. of ways $= \dfrac{11!}{2!\,2!\,2!}$.

2) No. of ways $= \dfrac{4!}{2!} \times \dfrac{8!}{2! \times 2!} = 3 \times 8!$

Circular Permutation

⤳ No. of circular arrangement of n different things equals to $\frac{1}{n} \times$ No. of linear arrangement of n different things.

$$= \frac{1}{n} \times n! = (n-1)!$$

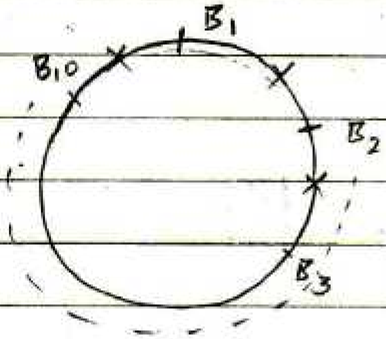


$\Longrightarrow$ No. of circular permutation $= (n-1)!$

⤳ When clockwise and anticlockwise arrangements are same, then no. of circular arrangement of n different things is equal to $\frac{(n-1)!}{2}$

Ques. find no. of ways in which 10 boys and 10 girls sit alternatively around a round table

Sol^n —



No. of ways = $10! \times 9!$

$10!$ → Arrangement of 10 boys b/w 10 gaps

$9!$ → Arrangement of 10 girls

Ques. No. of ways in which 10 different diamonds can be arranged to form a necklace.

Sol^n — No. of ways = $\frac{(10-1)!}{2} = \frac{9!}{2}$

NOTE :—

→ Clockwise and Anticlockwise arrangement are same in necklace, garland and different in when people sit around a round table

Ques. find the no. of ways in which 6 gentleman and 5 ladies be seated around a table so that every gentleman may have a lady by his side.

Sol^n — No. of ways = $2! \times {}^6P_6 = 2! \times 6! = 1440$.

JEE Mains - 2026
(28 Jan shift - 2)

Que. 8 persons enter a lift at ground floor and will go up to 10th floor. The no. of ways in which the 3 persons can exit the lift at different floor if the lift doesn't stop at 1st, 2nd and 3rd floor.

Sol^n - F_{10} —
F_9 —
F_8 —
F_6 —
F_5 —
F_4 —
F_3 ×
F_2 ×
F_1 ×

$$\text{No. of ways} = {}^7P_3 = \frac{7!}{4!}$$

$$= \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 30 \times 7 = 210.$$

JEE Mains - 2026
{28 Jan shift - 1}

Que. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Let x be no. of 9 - digit no. be formed using the digit of the set S. Such that only one digit is repeated exactly twice.
Let y be no. of 9 - digit no. formed from the digit of set S. such that only ~~twie~~ two digits are repeated and each of this repeated exactly twice. Then

1) $56x = 9y$ 2) $45x = 7y$
3) $21x = 4y$ 4) $29x = 5y$

Sol^n - for x

$$x = \underbrace{{}^{9}C_1 \times {}^{8}C_7}_{\text{Selection}} \times \underbrace{\frac{9!}{2!}}_{\text{Arrangement}}$$

$$\Rightarrow x = 9 \times 8 \times \frac{9!}{2!} \longrightarrow (I)$$

for y.

$$y = \underbrace{{}^{9}C_2 \times {}^{7}C_5}_{\text{Selection}} \times \underbrace{\frac{9!}{2!\,2!}}_{\text{Arrangement}}$$

$$y = \frac{9!}{2!\,\cancel{7!}} \times \frac{\cancel{7!}}{2!\,5!} \times \frac{9!}{2!\,2!} = \frac{9!}{2!}\left(\frac{9\times\cancel{8}^{\,2}\times 7\times\cancel{6}^{\,3}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}}\right)$$

$$y = 18 \times 21 \; \frac{9!}{2!} \longrightarrow (II)$$

On comparing from (I) & (II), we get

$$\Rightarrow \frac{x}{4} = \frac{y}{21} = 9 \cdot 9! \Rightarrow 21x = 4y.$$

JEE Mains - 2026

{24 Jan, Shift - 2}.

Ques. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of word "UDAYPUR".

a) 1581 b) 1578 c) 1579 d) 1580

Solⁿ ADPRUUY

A — — — — — — $\quad 6!/2! = 360$

D — — — — — — $\quad 6!/2! = 360$

P — — — — — — $6!/2! = 360$

R — — — — — — $6!/2! = 360$

U A — — — — — $5! = 120$

U D A P — — — $3! = 6$

U D A R — — — $3! = 6$

U D A U — — — $3! = 6$

U D A Y P R — $1! = 1$

U D A Y P U R $1! = 1$

Total no. of ways $= 4 \times 360 + 120 + 3 \times 6 + 2 \times 1$
$= 1440 + 138 + 2 = 1580$

JEE Mains - 2024
{24 Jan Shift - 1}

Ques. The number of number greater than 5000 and less than 9000 and divisible by 3 that can be formed using the digits 0, 1, 2, 5, 9 iff the repitition of the digits is allowed is —

Method-1.

Solⁿ — $a, b, c, d \in \{0, 1, 2, 5, 9\}$

$5000 < abcd < 9000$

$a+b+c+d = 3k$

$\downarrow$

$5+b+c+d = 3k$. No. of numbers

1. 5 1 0 0 ⟶ $3!/2! = 3$.

2. 5 9 1 0 ⟶ $3!/1 = 6$

3.

3. 5 0 2 5 ⟶ $3! = 6$

4. 5 0 5 5 ⟶ $3!/2! = 3$

5. 5 1 1 2 ⟶ $3!/2! = 3$

6.	5	0 2 2	$\longrightarrow$	3
7.	5	5 1 1	$\longrightarrow$	3
8.	5	9 2 2	$\longrightarrow$	3
9.	5	9 5 2	$\longrightarrow$	$3! = 6.$
10.	5	9 9 1	$\longrightarrow$	3
11.	5	5 5 9	$\longrightarrow$	3

$\therefore$ Sum $= 42$

$\downarrow$

Ans.

Method-2.

$a, b, c, d \in \{0, 1, 2, 5, 9\}$

$5000 < abcd < 9000$

$5 + b + c + d = 3k.$

$\downarrow$

$R\left(\frac{5}{3}\right) + R\left(\frac{b}{3}\right) + R\left(\frac{c}{3}\right) + R\left(\frac{d}{3}\right) = 3k_1$

$d = 3q + r$

$2 + R\left(\frac{b}{3}\right) + R\left(\frac{c}{3}\right) + R\left(\frac{d}{3}\right) = 3k_1 \Longrightarrow 3$ or $6.$

Since,

$R\left(\frac{0}{3} \text{ or } \frac{9}{3}\right) = 0$ 2 cases

$R\left(\frac{1}{3}\right) = 1$ 1 cases

$R\left(\frac{2}{3} \text{ or } \frac{5}{3}\right) = 2.$ 2 cases

Now,

$R\left(\frac{b}{3}\right) + R\left(\frac{c}{3}\right) + R\left(\frac{d}{3}\right) = 1$ or $4.$

1) $1 + 0 + 0 = 1.$

Possible No. of No. formed $= 1 \times 2 \times 2 \times 3 = 12$

2) $2 + 1 + 1 = 4$

$= 2 \times 1 \times 1 \times 3 = 6$

3) $2 + 2 + 0 = 4$

$= 2 \times 2 \times 2 \times 3 = 24$

$\Rightarrow 42$

Ans.

Solⁿ — PQRPQRSTUVP.

$$\left.\begin{array}{lll} 3 & \text{Times} & P \\ 2 & '' & Q \\ 2 & '' & R \\ 1 & '' & S \\ 1 & '' & T \\ 1 & '' & U \\ 1 & '' & V \end{array}\right\} \text{7 different letters.}$$

Case 1. All letters are different

No. of words $= {}^7P_4 = {}^7C_4 \times 4!$

$= \frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$

Case 2. $\underbrace{\text{3 letters alike}}_{P}$, 1 different

No. of words $= 1 \times {}^6C_1 \times \frac{4!}{3!} = 6 \times 4 = 24$

Case 3. 2 alike - one type , 2 alike - other type

No. of words $= {}^3C_2 \times \frac{4!}{2!\,2!} = 3 \times 6 = 18$

Case 4. 2 alike , 2 different

No. of words $= {}^3C_1 \times {}^6C_2 \times \frac{4!}{2!}$

$= 3 \times \frac{6!}{4!\,2!} \times \frac{4!}{2!} = 540$

Total no. of words $= 840 + 24 + 18 + 540$

$= 42 + 1380$

$= 1422$ ⟶ Ans.

Question for above Solⁿ :—

→ The no. of four letter word with or without meaning which can be formed from the letters of the word "PQRPQRSTUVP"

Combination

$\Downarrow$

Selection

Ex:— Make all possible combinations of two letters out of A, B, C, D.

AB	BC
AC	BD
AD	CD

$\longrightarrow$ 6 combinations

$\Downarrow$

$^4C_2$ = Combination or selection of 2 out of 4 different objects.

$$\frac{4!}{(4-2)!\,2!} = 6$$

$\rightarrow$ Number of all possible combinations of r different object out of n different objects $= {}^nC_r$

$= \binom{n}{r}$ or $C(n, r)$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

Properties of ^nC_r

$\rightarrow$ $^nC_r = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n!}{(n-r)!} \times \dfrac{1}{(r!)}$

$\rightarrow$ $^nC_r = {}^nP_r \times \dfrac{1}{r!}$

$\Rightarrow$ $C(n,r) = P(n,r)$

$\rightarrow$ $^nC_0 = \dfrac{n!}{(n-0)!\,0!} = 1.$ $\qquad$ $^nC_n = 1.$

$\Rightarrow$ $^nC_0 = {}^nC_n = 1$

$\rightarrow$ $^nC_1 = {}^nC_{n-1} = n$

Date :

Pascal's Mirror formula.

$$\underbrace{{}^nC_0,\ {}^nC_1,\ {}^nC_2,\ {}^nC_3,\ \ldots\ldots,\ {}^nC_r,\ \ldots\ {}^nC_n}_{(n+1)\text{ terms (Binomial coefficient)}}$$

$\text{Max}^m \leftarrow {}^nC_{(\frac{n}{2}+1)} \leftarrow$ n-even

${}^nC_{n/2} \leftrightarrow {}^nC_{n-n/2}$

${}^nC_{\left(\frac{n-1}{2}\right)} \leftrightarrow {}^nC_{\left(\frac{n+1}{2}\right)} \leftarrow$ n-odd

or, ${}^nC_{n-\left(\frac{n-1}{2}\right)}$

Max^m

${}^nC_3 \longleftrightarrow {}^nC_{n-3}$

${}^nC_2 \longleftrightarrow {}^nC_{n-2}$

${}^nC_1 \longleftrightarrow {}^nC_{n-1}$

${}^nC_0 \longleftrightarrow {}^nC_n$

Mirror

${}^nC_r = {}^nC_{n-r}$

$\hookrightarrow$ Binomial Coefficient

${}^nC_r$

$\longrightarrow n \in N,\ n > 0$

$\longrightarrow r \in W,\ r \geq 0$

$\longrightarrow n - r \geq 0$

Que find Domain and Range

$$f(x) = {}^{9-x^2}C_{x^2-1}$$

Sol^n —

$9 - x^2 > 0$

$(3-x)(3+x) > 0$

$-(x-3)(x+3) < 0$

$(x-1)(x+1) \geq 0$

$x \in (-\infty, -1] \cup [1, \infty)$

$x \in \{\dots, -3, -2, -1, 1, 2, 3, \dots\}$

$+ \quad - \quad +$; $-3 \quad 3$

$x \in (-3, 3)$

$x \in \{1, 2\}$

$n - r \geq 0$

$9 - x^2 - x^2 + 1 \geq 0$

$10 - 2x^2 \geq 0$

$5 - x^2 \geq 0 \Rightarrow (x-\sqrt{5})(x+\sqrt{5}) \leq 0$

$+ \quad - \quad +$; $-\sqrt{5} \quad \sqrt{5}$

$x \in [-\sqrt{5}, +\sqrt{5}]$

$x \in \{1, 2\}$

$\therefore x \in \{1, 2\}$

$y \in \{f(1), f(2)\} \Rightarrow y \in \{^8C_0, {}^5C_3\}$

If $^nC_x = {}^nC_y$

then, $x + y = n$.

Que. $^nC_7 = {}^nC_4$. find n.

Sol^n — $^nC_x = {}^nC_y \Rightarrow x + y = n$.

$7 + 4 = n$

$\Rightarrow n = 11$.

→ Pascal's Triangle formula.

Algebraic Expansion

$(x+1)^1 = 1x^1 + 1x^0$

$(x+1)^2 = 1x^2 + 2x + 1$

$(x+1)^3 = 1x^3 + 3x^2 + 3x + 1$

$(x+1)^4 = 1x^4 + 4x^3 + 6x^2 + 4x + 1$

↓

$(x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \dots + {}^nC_n x^0$

Decimal System

$$
\begin{array}{ccccccccccc}
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

→ Pascal's Mirror

Binomial Coefficient

Pascal's Triangle formula.

$$
\begin{array}{ccccccccc}
 & & & {}^1C_0 & & {}^1C_1 & & & \\
 & & {}^2C_0 & & {}^2C_1 & & {}^2C_2 & & \\
 & {}^3C_0 & & {}^3C_1 & & {}^3C_2 & & {}^3C_3 & \\
{}^4C_0 & & {}^4C_1 & & {}^4C_2 & & {}^4C_3 & & {}^4C_4
\end{array}
$$

→ Pascal's Mirror

Pascal's observation :—

→ ${}^1C_0 + {}^1C_1 = {}^2C_1$

→ ${}^2C_0 + {}^2C_1 = {}^3C_1$

→ ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

Or,

${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

Or,

${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r$

Ex:– $^{47}C_4 + \sum_{j=1}^{5} \left(^{52-j}C_3\right)$

find value.

Sol^n:– $^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4 = {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$

→ $^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

→ $\dfrac{^nC_{r-1}}{^{n-1}C_{r-1}} = \dfrac{n}{n-r+1}$

→ Maximum value of ^nC_r $\quad 0 \le r \le n$, $r \in W$

$n \longrightarrow$ even

Greatest value of $^nC_r = {}^nC_{\frac{n}{2}+1}$

$n \longrightarrow$ odd

Greatest value of $^nC_r = {}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$.

Ques 3 gentlemens and 3 ladies are candidate for two vacancies. A voter has to vote two candidates. If how many can one cast his vote?

Sol^n:– No. of ways $= {}^6C_2$

Ques. In how many ways can a cricket 11 be choosen of out of batch of 15 players. if: I) There is no restriction on the selection.

II) A particular player is always choosen.

III) A particular player is never choosen.

Solⁿ—
1) ${}^{15}C_{11}$
2) ${}^{15-1}C_{11-1} = {}^{14}C_{10}$
3) ${}^{15-1}C_{11} = {}^{14}C_{11}$

Ques. A committee of 12 is to formed from 9 women and 8 men. In how many ways this can be done. If atleast 5 women have to included in a committee. In how many ways these ~~mom~~ committee
a) The womens are in majority
b) The men are in majority

Solⁿ—
12 ⟶ Committee
9 women + 8 men

No. of ways $= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \cdot {}^8C_5 + {}^9C_8 \cdot {}^8C_4 + {}^9C_9 \cdot {}^8C_3$.

a) No. of ways $= {}^9C_7 \times {}^8C_5 + {}^9C_8 \cdot {}^8C_4 + {}^9C_9 \times {}^8C_3$.

b) No. of ways $= {}^8C_8 \times {}^9C_4 + {}^9C_5 \times {}^8C_7$

Ques. A committee of 3 person is to be constituted from a group of 2 men and 3 women. In how many ways can this be done. How many of these committee would consist of 1 men and 2 women.

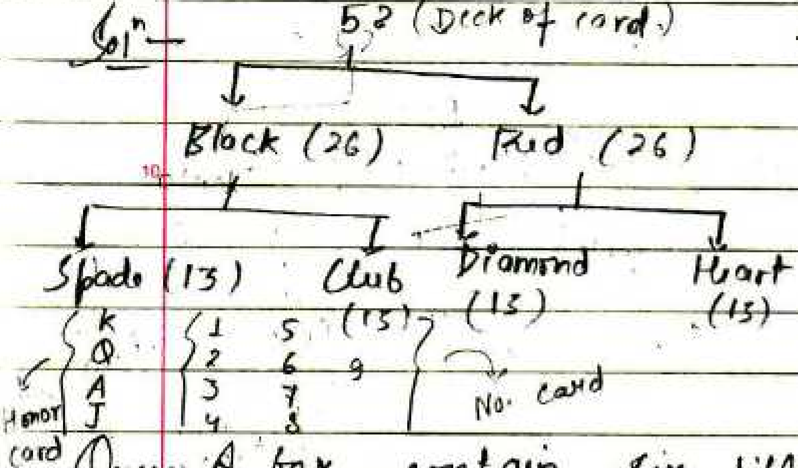
Solⁿ— 1) No. of ways $= {}^3C_1 \times {}^2C_2 + {}^3C_2 \times {}^2C_1 + {}^3C_3 \cdot {}^2C_0$
$= 3 \times 1 + 3 \times 2 + 1 = 10$

2) No. of ways $= {}^2C_1 \times {}^3C_2 = 6$

Ques. What is the no. of ways of choosing of 4 cards from the pack of 52 cards? In how many of

there :- 1. 4 cards are of same suit
2. 4 cards belong to four different suits.
3. 4 cards are face cards.
4. 2 are red card and 2 are black card.
5. Card are of the same colour.

Sol^n —



→) $^{52}C_4$

1) $4 \cdot {}^{13}C_4$

2) $^{13}C_1 \times 4$

3) $^{12}C_4$

4) $^{26}C_2 \times {}^{26}C_2$

5) $2 \cdot {}^{26}C_4$

Que. A box contain six different red ball and 5 different white balls. In how many ways can six balls be selected so that there are atleast two ball of each colour.

Sol^n —

6 — red ball
5 — white ball

6 — ball select

$$\text{No. of ways} = {}^6C_4 \cdot {}^5C_2 + {}^5C_3 \cdot {}^6C_3 + {}^5C_4 \cdot {}^6C_2$$

$$= \frac{6!}{4!2!} \cdot \frac{5!}{3!2!} + \frac{5!}{2!3!} \cdot \frac{6!}{3!3!} + \frac{5!}{4!1!} + \frac{6!}{4!2!}$$

$$= \frac{6!}{4!2!}\left(\frac{5 \times 4}{2} + 5\right)$$

$$= \frac{6!}{4!2!} \times 15 + \frac{5 \times 4^2}{2} \cdot \frac{6 \times 5 \times 4}{6}$$

$$= \frac{{}^3 6 \times 5}{2} \times 15 + 10 \times 20 = 225 + 200$$

$$= 425$$

Que. for the post of 5 teachers, there are 23 applicant, 2 post are reserved for SC candidate and there are 7 SC candidate among the applicant. In how many can the selection be made.

Sol^n — 5 teachers

23 applicant

7 SC   2 SC reserved.

$\Rightarrow$ No. of ways $= {}^7C_2 \times {}^{16}C_3$

$$= \frac{7!}{5!\,2!} \times \frac{16!}{13!\,3!} = \frac{7\times\cancel{6}}{2} \times \frac{\overset{8}{\cancel{16}}\times 15\times 14}{\cancel{6}}$$

$$= 56\times15\times14 = 210\times56 = 11760$$

Ques. How many triangle can be formed by joining the vertix of hexagon.

Sol^n — No. of Δ formed $= {}^6C_3 = \frac{6!}{3!\,3!} = \frac{6\times5\times4}{\cancel{3\times2}} = 20$

NOTE :—

→ n different points are given in a plane such that no 3 points are collinear, then

no. of line segments formed. $= {}^nC_2$

No. of triangle formed $= {}^nC_3$.

Ques. A polygon has 44 diagonals. find the no. of sides of the diagonals.

→ In a n-sided polygon

No. of diagonal $= {}^nC_2 - n = \frac{n(n-1)}{2} - n$

$$= \frac{n(n-3)}{2}$$

Sol^n —

$${}^nC_2 - n = 44$$

$$n(n-3) = 88$$

$$n(n-3) = 11\times 8 \Rightarrow n = 11$$

Restricted Combination

→ No. of combinations of n different things taken r at a time when k no. of particular things must be included $= {}^{n-k}C_{r-k}$.

→ No. of combinations of n different things taken r at a time when k no. of particular things must be excluded $= {}^{n-k}C_{r}$.

Ques. A father with 8 children takes 3 at a time to Zoological park with taking the same three children more than once. Then find the no. of time each child will go.

Sol^n — No. of ways each child will go $= {}^{8-1}C_{3-1} = {}^{7}C_{2} = 21$

Ques. Find the no. of six digit no.s using 1, 2, 3, 4, 5 such that any digit that appears at ~~twice~~ atleast in the number appears twice.

Sol^n — Case 1. :— Digits appear in number appears twice.

Ex :— 1 1 2 2 3 3.

No. of no. $= {}^{5}C_{3} \cdot \frac{6!}{2!2!2!} = 10 \times 90 = 900$

Case 2. :— Digits appear in the number appear 3 times

Ex :— 1 1 1 2 2 2

No. of no. $= {}^{5}C_{2} \times \frac{6!}{3!3!} = 200 = 10 \times 20$

Case 3. :— Digits appear in the number appears 4 time alike, 2 time alike.

Ex:— 1 1 1 1 2 2

No. of no. $= 2! \times {}^5C_2 \times \frac{6!}{4!\,2!} = 2 \times 10 \times 15 = 300$

Case 4:— Digit appears in the number appears 6 times

Ex:— 1 1 1 1 1 1

No. of no. $= {}^5C_1 \times \frac{6!}{6!} = 5$

$\therefore$ No. of such number formed = 1405

Chess Board Problems

Que find no. of rectangles in the chess board having 9 horizontal and 9 vertical lines.

Sol:— No. of rectangles $= {}^9C_2 \times {}^9C_2$

Total no. of square using equalling spacing of 9 Horizontal and 9 vertical $= 8 \times 8 + 7 \times 7 + 6 \times 6 + 5 \times 5 + 4 \times 4 + 3 \times 3 + 2 \times 2 + 1 \times 1$

$= 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2$

$= \frac{9 \times 8 \times 17}{6} = 204.$

funda :—

No. of Horizontal & vertical lines = n.

No. of rectangles $= {}^nC_2 \times {}^nC_2$

No. of Squares $= \sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + \dots + n^2.$

→ Points in a plane (including collinear points)

No. of lines joining n points in a plane where r points are collinear ; $n \geq r$

No. of lines is equal to the no. of ways in which two points can be selected is $^nC_2$ but r points are collinear.
So, $^rC_2$ - no. of lines lie on the same line.
Therefore,
No. of different lines can be formed is $^nC_2 - {}^rC_2 + 1$.

Ques Let the eleven letters A, B, ..., k denote an arbitary permutation of the integers (1, 2, ..., 11). Then prove that (A-1) (B-2) (C-3) .....(k-11) is always even.

Sol:- A = No. of permutation of integer taking 1 at a time
$= {}^{11}P_1 = 11$

B = No. of permutation of integer taken 2 at a time $= {}^{11}P_2 = 110$.

C = No. of permutation of integer taken 3 at a time
$= {}^{11}P_3 = 11 \times 10 \times 9$.

|

k = No. of permutation of integer taken all at a time $= {}^{11}P_{11} = 11!$

$\Rightarrow (A-1)\,(B-2)\,(C-3)\ldots(k-11)$
$= (11-1)\,(110-2)\,(C-3)\;\ldots\;(k-11)$
$= 10 \times \ldots\ldots$ = Always even.

Ques. find the no. of different words that can be formed using all letters of the words, 'SHASHANK' such that in any word, the vowels are separated, by at least two consonants.

Solⁿ— Case 1: Between two A if we have only 2 consonants

A _ _ A _ _ _ _

_ A _ _ A _ _ _

_ _ A _ _ A _ _

_ _ _ A _ _ A _

_ _ _ _ A _ _ A

No. of words $= 5 \times \frac{6!}{2!2!}$

Case 2. 3 Gap.

A _ _ _ A _ _ _

_ A _ _ _ A _ _

_ _ A _ _ _ A _

_ _ _ A _ _ _ A

No. of words $= 4 \times \frac{6!}{2!2!}$

Case 3. 4 Gaps

No. of words $= 3 \times \frac{6!}{2!2!}$

Case 4. 5 Gaps.

No. of words $= 2 \times \frac{6!}{2!2!}$

Case 5. 6 Gaps

No. of words $= 1 \times \frac{6!}{2!2!}$

$\therefore$ Total No. of words formed $= \frac{6!}{2!2!}(5+4+3+2+1)$

$= 2700$

Que. find the number of permutations of the letters of the word 'HINDUSTAN' such that none of the patterns HIN, DUS and TAN appears.

Solⁿ— $n(U) = \frac{9!}{2!}$ ⟶ HINDUSTAN

$$\underset{A}{\underset{\downarrow}{\text{HIN}}} \quad \underset{B}{\underset{\downarrow}{\text{DUS}}} \quad \underset{C}{\underset{\downarrow}{\text{TAN}}}$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

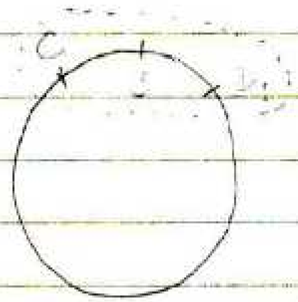
$$= 7! + \frac{7!}{5!} + 7! - 5! - 5! - 5! + 3!$$

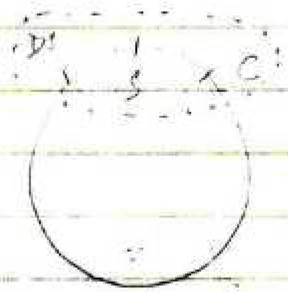
$$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$$

$$= \frac{9!}{2!} - \left[\frac{5}{2} \times 7! - 3 \times 5! + 3!\right]$$

Que In how many ways can 15 numbers of a council sit around a circular table, when the secretary is to sit on one side of the chairman and the Deputy Secretary on the ~~or~~ other side?

Sol –





∴ No of ways = 12!   ∴ No. of ways = 12!

∴ Total no. of ways = 2 × 12!

Ques A shelf contains 20 different books of which 4 are in single volume and the others form sets of 8, 5 and 3 volume of each set are together and in their due order.

Sol – 20 Book

4 in single Volume

8 —

5 —

3 —

∴ No. of ways = (4+3)!
= 7!

Total no. of Combinations of "n" different object when taken one or more time

$$f(x) = (1+x)^n = {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \dots + {}^nC_n x^n$$

At $x = 1$

$$f(1) = 2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \dots + {}^nC_n$$
$$= \sum_{r=0}^{n} {}^nC_r = \sum_{r=0}^{n} C(n,r) = \sum_{r=0}^{n} C_r = \sum_{r=0}^{n} \binom{n}{r}$$

Total no. of combination of "n" different objects taken 1 or more at a time.

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$
$$= ({}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n) - {}^nC_0$$
$$= 2^n - 1$$

Or,

for each object there are two choices either selection or rejection.

Total no. of combination $= \underbrace{2 \times 2 \times 2 \dots \times 2}_{n \text{ time}}$

(0 or more) $= 2^n$

Total no. of combination of n different object taken 1 or more at a time $= 2^n - 1$.

Total no. of selection of one or more things from 'p' identical of one type, 'q' identical of other type, 'r' identical of third type and 'n' different things.

Ex:- 3 identical object
No. of combination of 0 or more objects = ?
→ No. of ways of selection of zero objects = 1
→ " " " " " 1 " = 1
→ " " " " " 2 " = 1
→ " " " " " 3 " = 1

No. of combination of 0 or more objects out of 'p' identical objects = $p+1$
No. of combination of 0 or more objects out of 'q' identical objects = $q+1$
No. of combination of 0 or more objects out of 'r' identical objects = $r+1$
No. of combination of 0 or more objects out of 'n' different objects = 2^n.

→ No. of combinations of 0 or more objects out of 'p' identical, 'q' identical, 'r' identical & 'n' different objects = $(p+1)(q+1)(r+1)2^n$.

Ques. How many ways we can select two or more people out of 20 people.

Solⁿ- No. of ways = $2^{20}-20-1 = 2^{20}-21$

Ques. find total no. of ways of selecting one or more things out of given 4 identical of one type, 5 identical of other type.

Solⁿ – No. of ways = $(5+1)(4+1) - 1 = 6\times5-1 = 30-1 = 29$.

Ques. find the no. of possible combinations of a given 8 identical of one type, 3 identical of other type, 9 identical of third type and 20 different things.

Solⁿ – No. of possible combinations $= (8+1)(9+1)(3+1)\cdot 2^{20}$

$= 9\times10\times4\times2^{20}$

$= 360\times2^{20}$

No. of Divisor & Sum of divisor of a Natural Number

Ex :– $6 = 2^1\times3^1 \longrightarrow (1+1)(1+1)$

$= 2\times2 = 4.$

1, 2, 3, 6

Sum of divisor $= 1+2+3+6 = 12$.

Funda

$N = P_1^{\alpha_1}\times P_2^{\alpha_2}\times P_3^{\alpha_3}\times\cdots\cdots\times P_n^{\alpha_n}$

No. of divisor = Combinations of 0 or more primes out of α_1 identical prime P_1

α_2 " " P_2

α_3 " " P_3

⋮

α_n " " P_n

$= (\alpha_1+1)(\alpha_2+1)\cdots\cdots(\alpha_n+1)$

No. of divisor = No. of terms in the expansion of

$$\underbrace{(P_1^0+P_1^1+\cdots P_1^{\alpha_1})}_{\alpha_1+1}\underbrace{(P_2^0+P_2^1+P_2^2+\cdots\cdots+P_2^{\alpha_2})}_{\alpha_2+1}\cdots\cdots\underbrace{(P_n^0+P_n^1+P_n^2+\cdots P_n^{\alpha_n})}_{\alpha_n+1}$$

$$= (\alpha_1+1)(\alpha_2+1) \quad\quad (\alpha_n+1)$$

Sum of divisor $= (P_1^0 + P_1^1 + P_2^2 + P_1^3 + \cdots + P_1^{\alpha_1})$
$(P_2^0 + P_2^1 + P_2^2 + \cdots + P_2^{\alpha_2}) \cdots\cdots$
$(P_n^0 + P_n^1 + P_n^2 + \cdots + P_n^{\alpha_n})$

$$= \left(\frac{P_1^{\alpha_1+1}-1}{P_1-1}\right)\left(\frac{P_2^{\alpha_2+1}-1}{P_2-1}\right)\cdots\cdots\left(\frac{P_n^{\alpha_n+1}-1}{P_n-1}\right)$$

Que. find no. of divisor and Sum of divisor of 248.

Sol^n–

$$\begin{array}{r|l} 2 & 248 \\ \hline 2 & 124 \\ \hline 2 & 62 \\ \hline & 31 \end{array}$$

$248 = 31^1 \times 2^3$.

No. of divisor $= (3+1)(1+1) = 4\times 2 = 8$

Sum of divisor $= (1+2+4+8)(1+31)$
$= 32 \times 15 = 480$.

Que. i) No. of divisor of $N = 3^P \cdot 6^m \cdot 21^n$ is ?

Sol^n $N = 3^P \cdot 6^m \cdot 21^n$.
$= 3^{P+m+n} \cdot 2^m \cdot 7^n$.

No. of divisor $= (m+1)(n+1)(P+m+n+1)$

ii) No. of odd proper divisor of N
$= (P+m+n+1)(n+1) - 1$

$N' = 3^{P+m+n} \cdot 7^n$ { as it is odd }

→ No. of proper of divisor $= (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_n+1) - 1$

→ Proper divisor of 'N' are all the divisors except 'N'.

Ques. No. of divisor of $14^5 \times 9^8 \times 7^9 = ?$

Sol^n –

$$14^5 \cdot 9^8 \cdot 7^9 = 2^5 \cdot 7^5 \cdot 3^{2\times 8} \cdot 7^9$$
$$= 2^5 \cdot 3^{16} \cdot 7^{14}.$$

No. of divisor $= 6 \times 17 \times 15$
$= 90 \times 17 = 1530.$

Division & Distribution of Different object

Ex: — 5 objects (Different type)

↓

two groups.

↙ ↘

3. 2.

In how many ways, we can divide 5 into two groups of size 3 & 2.

No. of division $= {}^5C_2$ or ${}^5C_3 = \dfrac{5!}{2!\,3!}$

funda.

No. of ways of $(m+n)$ different objects divided into two groups of the size (m) & (n) equal to ${}^{m+n}C_m$

$m+n$

↙ ↘

m n

No. of division $= {}^{m+n}C_n$ Or, ${}^{m+n}C_m = \dfrac{(m+n)!}{m!\,n!}$

Division + Arrangement = Distribution

No. of ways of distributing $(m+n)$ different objects between two person if one get (m) objects and other get (n) object

= No. of ways of division of $(m+n)$ object into two group × No. of ways of these two groups can be distributed into two persons.

$$= \frac{(m+n)!}{m!\,n!} \times 2!$$

NOTE :—

1\. for $(m+n+p)$ different objects

$$(m+n+p)$$

$$\{(m) \quad (n) \quad (p)\} \longrightarrow \text{Three groups.}$$

No. of ways of division $\longrightarrow \dfrac{(m+n+p)!}{m!\,n!\,p!}$

No. of ways of distribution $\longrightarrow \dfrac{(m+n+p)!}{m!\cdot n!\,p!} \times 3!$

2\. for $(x_1+x_2+x_3+\cdots+x_n)$ different objects.

$$(x_1+x_2+x_3+\cdots\cdots+x_n)$$

$$\{(x_1) \quad (x_2) \quad (x_3) \quad (x_4) \ldots (x_n)\} \longrightarrow n \text{ groups.}$$

No. of ways of division $\longrightarrow \dfrac{(x_1+x_2+\cdots\cdots+x_n)!}{x_1!\cdot x_2!\cdots\cdots x_n!}$

No. of ways of distributing (n) groups among (n) persons $\longrightarrow \dfrac{(x_1+x_2+\cdots+x_n)!}{x_1!\cdot x_2!\cdots\cdots x_n!} \times n!$

Qus. In how manys 10 different objects can be distributed among 3 children So, that each children receives atleast two object. The no. of object recived by each child is inversely proportional to its age.

Sol^n^ –

10 object

5   3   2

$C_1 < C_2 < C_3$

No. of ways of Division $= \dfrac{10!}{5!\,3!\,2!}$

Qus. 5 different object are to be distributed among 3 person Such that no two person get the same no. of object. find the no. of ways this can be done.

Sol^n^ – 5 object

1) $\underline{0}$ $\underline{2}$ $\underline{3}$ $\longrightarrow \dfrac{5!}{0!\,2!\,3!} \times 3!$

2) $\underline{0}$ $\underline{1}$ $\underline{4}$ $\longrightarrow \dfrac{5!}{1!\,0!\,4!} \times 3!$

$$\frac{5!}{2!} + \frac{5!}{4!} = 5 \times 4 \times 3 + 5 \times 6$$

$$= 60 + 30 = 90$$

Division of object into group of equal size

Ex :— 4 different letters
a, b, c, d

If we use the formula.
No. of ways of division $= \frac{4!}{2!\,2!} = 6$

Group 1	Group 2
ab	cd
ac	bd
ad	bc
bc	ad
bd	ac
cd	ab

same

Actual No. of ways of division $= 3 = \frac{\frac{4!}{2!\,2!}}{2!}$

No. of ways of distribution the 2 groups into two person $= \frac{4!}{2!\,2!\,2!} \times 2! = \frac{4!}{2!\,2!}$

1.

→ No. of ways of division (2n) objects into two groups of equal size $= \frac{(2n)!}{n!\,n!\,2!}$

→ No. of ways of distributing 2n objects in two groups of equal size to two person $= \frac{(2n)!}{n!\,n!\,2!} \times 2!$

2.

→ No. of ways of division of 3n objects into 3 groups of equal size $= \frac{(3n)!}{n!\,n!\,n!\,3!}$

→ No. of ways of distributing 3n objects in 3 groups of equal size to three person $= \frac{(3n)!}{n!\,n!\,n!\,3!} \times 3!$

Division of different objects into multiple group.

Ques find no. of ways of division of 12n object into 5 groups of size 2n, 2n, 2n, 3n, 3n. Also find no. of ways of distributions of these group into 5 persons.

Sol^n —

$$\text{No. of ways} = \frac{\dfrac{(12n)!}{(2n)!\,(2n)!\,(2n)!\,(3n)!\,(3n)!}}{2!\times 3!} \times 5!$$

$$= \frac{(12n)! \times 2! \times 3!}{(2n)!\,(2n)!\,(2n)!\,(3n)!\,(3n)! \times 5!}$$

Distribution of 'n' distinct objects in 'r' different boxes if in any box, Any Number of Object are placed (Empty Boxes are Allowed)

Ex :— 1) O_1 ; O_2

$\boxed{}$ B_1

find no. of ways of distributing 2 different object in one box (Box may have any no. of object)

$= 1 \longrightarrow \boxed{O_1 ; O_2}$

2) O_1 ; O_2

$\boxed{}$ B_1 $\boxed{}$ B_2

$= 4 \longrightarrow$

B_1	B_2	B_1	B_2
$\boxed{}$	$\boxed{O_1\ O_2}$	$\boxed{O_1}$	$\boxed{O_2}$
$\boxed{O_1\ O_2}$	$\boxed{}$	$\boxed{O_2}$	$\boxed{O_1}$

→ Distribution of n different object in r different boxes if any box may contain any number of object $= \underbrace{r \times r \cdots \times r}_{n\text{ times}}$

$= r^n$

Here, n objects have r choice ⇒ No. of ways $= r^n$

Que Distribute 2 object into 3 boxes.

→ No. of distribution = $3^2 = 3 \times 3 = 9$

Special Cases,

1. → If any set contain "n" element, then no. of subset $= 2^n$

REASON :—

Since, for each element, there always two choices, (Exclusion / Inclusion)

So, Total no. of subset $= \underbrace{2 \times 2 \times 2 \cdots \times 2}_{n \text{ times}} = 2^n$

2. → Distribution of 'n' object among two boxes if box may contain any number of object (Empty boxes are allowed). $= 2^n$

3. → Distribution of 'n' object among three boxes if box may contain any no. of object (Empty boxes are allowed) $= 3^n$.

Distribution of 'n' distinct object into r different boxes if empty boxes are not allowed. Or, In each box at least one object must be kept.

Ex :— 1) Distribution 2 obj, in 1 box with above said condition

$O_1 ; O_2$ $\qquad = 1$.

B_1

2) Distribute 2 obj into 2 diff. boxes.

$O_1 ; O_2$, B_1 ; B_2 $\qquad = 2$.

Principle of inclusion & Exclusion

Funda.

$$n(A \cup B \cup C) = \underbrace{n(A) + n(B) + n(C)}_{\text{Inclusion}} - \underbrace{[n(A \cap B) + n(A \cap C) + n(B \cap C)]}_{\text{Exclusion}} + \underbrace{n(A \cap B \cap C)}_{\text{Inclusion}}$$

→ No. of distribution of 2 objects into 2 boxes when no boxes are empty $= 2^2 - {}^2C_1(2-1)^2$
$= 4 - 2 = 2.$

→ No. of ways of distribution of 'n' different object into 'r' distinct boxes if empty boxes are not allowed or A box must have atleast one object $= r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - {}^rC_3(r-3)^n + {}^rC_4(r-4)^n \ldots\ldots + (-1)^{r-1}\,{}^rC_{r-1}(r-(r-1))^n$

next

Division and Distribution of identical objects

* Distribution of 'n' identical objects in 'r' different boxes if empty boxes are allowed.

Ex :- Distribute 3 identical objects in '2' different boxes if empty boxes are allowed.

1. $\boxed{}_{B_1}$ $\boxed{0\,0\,0}_{B_2}$ 2. $\boxed{0\,0\,0}_{B_1}$ $\boxed{}_{B_2}$

3. $\boxed{0\,0}_{B_1}$ $\boxed{0}_{B_2}$ 4. $\boxed{0}_{B_1}$ $\boxed{0\,0}_{B_2}$

4 no. of ways.

* Beggar's concept

Ex :- 3 coins are to be distributed in two beggars. find the no. of ways if a beggar can get no coin.

We have to distribute 3 coins in 2 beggars.

→ We have to find + no. of ways of 2 groups of 3 coins.

O O O '|'

So, we keep 3 coins and one partition line (since two beggars so (2-1) = 1 partition line) in a row.

So, the required no. of ways to distribute 3 coins in two beggars is equivalent to arranging 3 identical coin and (2-1) = 1 partition $= {}^{3+2-1}C_{2-1} = {}^{4}C_{1} = 4$.

Ques. find the no. of ways of distributing of 20 identical apple to Mr. Sarvesh and their 10 students if every one may get any no. of apple.

Sol:- No. of ways $= {}^{20+11-1}C_{11-1} = {}^{31-1}C_{11-1} = {}^{30}C_{10}$.

$\underbrace{0\ 0\ 0\ 0\ 0 \dots\ 0}_{20 \text{ Apples}} \qquad \underbrace{|\ |\ |\ |\ \dots\dots\ |}_{(11-1) \text{ partition}}$

→ $\underbrace{0\ 0\ 0\ 0 \dots\dots\ 0}_{n \text{ identical objects}} \qquad \underbrace{|\ |\ |\ |\ |\ \dots\dots\ |}_{(r-1) \text{ partition line}}$

Distribution of 'n' identical objects in 'r' different boxes if empty boxes are allowed.

No. of ways $= {}^{n+r-1}C_{r-1}$ or, ${}^{n+r-1}C_{n}$

→ No. of non negative integral solution set

$x_1 + x_2 + x_3 + \dots + x_r = n$.

$0 \le x_1, x_2, x_3 \ldots\ldots, x_r \le n$

$; \ x_1, x_2, x_3 \ldots\ldots, x_r \in I$

No. of non negative integral solution $= {}^{n+r-1}C_{r-1}$

Or, ${}^{n+r-1}C_n$

Ex :— $\underbrace{x+y+z}_{\text{3 beggar}} = \underbrace{3}_{\text{3 coin}}$

Any beggar may get any no. of coin.

No. of non negative solution $= {}^{3+3-1}C_{3-1} = {}^5C_2$.

★ Distribution of n identical objects into r different boxes if Empty boxes are not allowed.

Or, a box must have atleast one object.

$\underbrace{O - O - O - O - \cdots\cdots\cdots O - O}_{(n-1) \text{ gaps available}}$

$\left\{ \begin{array}{c} n \text{ coins} \\ \downarrow \\ r\text{-beggars.} \end{array} \right.$

Selection of $(r-1)$ partition line out of $(n-1)$ availables.

No. of ways $= {}^{n-1}C_{r-1}$

→ No. of positive integral solution in $x_1 + x_2 + \cdots\cdots + x_r = n$ $= {}^{n-1}C_{r-1}$

$1 \le x_1, x_2, \ldots\ldots, x_r \le$ [illegible]

$x_1, x_2, \ldots\ldots, x_r \in N$

Que. find the number of ways in which 13 identical ~~objects~~ apples can be distributed among 3 persons so that no two persons recive equal number of apples and each one can recive any number of apples.

Solⁿ – Total no. of ways of distributing apples $= {}^{13+3-1}C_{3-1}$

$= {}^{15}C_2 = 105$

No. of ways when exactly two persons receive equal no. of apples $= 7 \times \frac{3!}{2!} = 21$

0	0	13	2	2	9	4	4	5	6	6	1
1	1	11	3	3	7	5	5	3			

$\Rightarrow$ Req. no. of ways $= 105 - 21 = 84$ ways.

Que. Find the no. of ways in which 15 identical apples and 10 identical oranges can be distributed among three persons, each receiving none, one or more.

Solⁿ – Total no. of ways $= {}^{15+3-1}C_{15} \times {}^{10+3-1}C_{10}$

$= {}^{17}C_2 \times {}^{12}C_2 = \frac{17 \times 16}{2} \times \frac{12 \times 11}{2}$

$= 8976.$

Que. Find the no. of non-negative integral solⁿ of the eqⁿ $x+y+z+2w = 20$.

Solⁿ $x+y+z+2w = 20$

$0 \le 2w \le 20 \longrightarrow 0 \le w \le 10 \qquad w \in I.$

$w=0 \qquad x+y+z = 20 \qquad {}^{20+3-1}C_{3-1} = {}^{22}C_2$

$w=1 \qquad x+y+z = 18 \qquad {}^{18+3-1}C_{3-1} = {}^{20}C_2.$

$w=2 \qquad x+y+z = 16 \qquad {}^{16+3-1}C_{3-1} = {}^{18}C_2.$

$\vdots \qquad \vdots \qquad \vdots$

$w=10 \qquad x+y+z = 0 \qquad {}^{0+3-1}C_{3-1} = {}^{2}C_2$

$\therefore$ No. of solutions $= {}^{22}C_2 + {}^{20}C_2 + {}^{18}C_2 + \dots + {}^{2}C_2$

$= 231 + 190 + 153 + \dots + 28 + 15 + 6 + 1$

Que. Find the non-negative integral solution.

$x+y+z+w \le 20.$

Sol^n — Method 1. No. of whole no. of sol^n

1. $x+y+z+w=0 \longrightarrow = {}^3C_3$
2. $x+y+z+w=1 \longrightarrow = {}^4C_3$
3. $x+y+z+w=2 \longrightarrow = {}^5C_3$

4. $x+y+z+w=20 \longrightarrow = {}^{23}C_3$

Total no. of ways $= {}^3C_3 + {}^4C_3 + {}^5C_3 + \dots + {}^{23}C_3$

$= {}^4C_4 + {}^4C_3 + {}^5C_3 + \dots + {}^{23}C_3$

${}^5C_4 +$

${}^5C_4$

${}^6C_4$

$= {}^{24}C_4$

Method 2.

$x+y+w+z \le 20$

$\hookrightarrow x+y+w+z+t = 20$

t — dummy Variable

No. of ways $= {}^{20+5-1}C_{5-1} = {}^{24}C_4$.

Qus. $x+y+z=30$

$2 \le x,\ 0 \le y, z$.

$x, y, z \in I$

find total possible solution

Sol^n $0 \le x-2$

$0 \le t \longrightarrow t = x-2$

$x+y+z-2 = 30-2$

$t+y+z = 28$

$0 \le t, y, z \le 28$; $t, y, z \in I$

No. of possible solution $= {}^{28+3-1}C_{3-1} = {}^{30}C_2$

Ques. $x+y+z+t=8$

$1 \le x, y, z, t \le 8$, $x, y, t, z \in N$

find all possible (x, y, z, t)

Solⁿ:- $1 \le x \le 8 \Rightarrow 0 \le x-1 \le 7$

$1 \le y \le 8 \Rightarrow 0 \le y-1 \le 7$

$1 \le z \le 8 \Rightarrow 0 \le z-1 \le 7$

$1 \le t \le 8 \Rightarrow 0 \le t-1 \le 7$

$(x-1)+(y-1)+(z-1)+(t-1)=4$

$a + b + c + d = 4$

No. of ways $= {}^{4+4-1}C_{4-1} = {}^{7}C_{3}$

Pigeon hole Principle (PHP)

If have 'n' items and you put them into 'm' containers $(n > m)$, then atleast one container must contain more than one item.

Least no. of item $= \left(\frac{n}{m}\right)$ $\quad (\cdot)$ - L.I.F

$\therefore$ No. of ways of distribution $= m^n$.

Refer to distribution of 'n' diff. object into 'r' different boxes (empty boxes are allowed).

Ques. You have 10 pigeon but only 9 pigeon holes. Atleast one hole are going to be crowded find atleast two bird find no. of ways.

Solⁿ:- If different 10 pigeon has 9 choices of hole then atleast one hole will have atleast two pigeon if we consider no hole has two or more pigeon then hole must have 0 or more pigeon per hole. that is not possible.

$\therefore$ No. of ways $= 9^{10}$

Ques. Distribute 9 different holes into 10 identical pigeons. When it is known that 1. Some hole may empty. 2. No hole are empty

Sol:- 1. ${}^{10+9-1}C_{9-1} = {}^{18}C_8$

2. ${}^{10-1}C_{9-1} = {}^{9}C_8$

Paper Discussion – 22/06/25

1. The value of $(2\cdot{}^1P_0 - 3\,{}^2P_1 + 4\,{}^3P_2 - \cdots$ up to 51^{th} term) + $1! - 2! + 3! - \cdots$ up to 51^{th} term) is equal to.

a) $1 - 51(51)!$ c) $1+(51)!$

b) $1+(52)!$ ✓ d) 1

$\longrightarrow \left(2\cdot 1! - 3\cdot 2! + 4\cdot 3! - 5\cdot 4! \cdots\cdots + 52\cdot 51!\right)$

$+ \left(1 - 2! + 3! - \cdots\cdots + 51!\right)$

$= \left(\cancel{2!} - \cancel{3!} + \cancel{4!} - \cancel{5!} + \cancel{6!} + \cdots + 52!\right) +$

$\left(1 - \cancel{2!} + \cancel{3!} - \cdots\cdots + \cancel{51!}\right)$

$= 1 + 52!$

(b) → Ans.

3. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is $336k$, then k is equal to

a) 8 ✓ b) 7 c) 4 d) 6.

$\longrightarrow \{0,1,2,3,4,5,6,7,8,9\}$

$\underbrace{\_\ \ \_\ \ \_\ \ \underset{2}{\_}\ \ \_}_{\text{5-digit number}}$

No. of number $= 8\times 8\times 7\times 1\times 6$

$= 336\times 8$

$= 336\times k.$

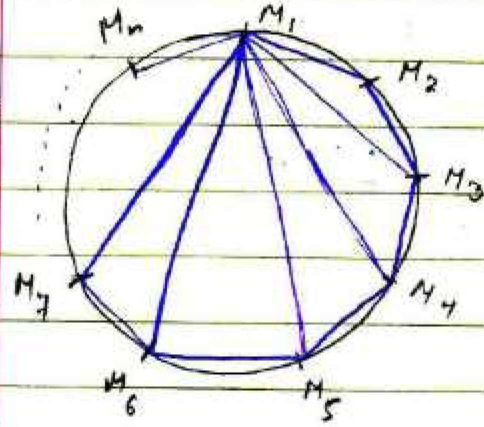
$\therefore k = 8.$

6. Let $n > 2$ be an integer. Suppose that there are 'n' metro stations in a city located along a circular path. Each pair of stations is connected by a straight track.

only. further, each pair of nearest stations is connected by blue lines, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n.

Sol^n – $n > 2$

$n \to$ Metro stations



$\longrightarrow$ Red line

$= {}^nC_2 - n$

$\longrightarrow$ Blue line

$= n$.

$${}^nC_2 - n = 99n$$

$${}^nC_2 = 100n$$

$$\frac{n!}{(n-2)!\,2!} = 100n$$

$$\frac{\cancel{n}(n-1)}{2} = 100\cancel{n} \Rightarrow n-1 = 200$$

$$\Rightarrow n = 201.$$

12. If ${}^{20}C_1 + 2^2({}^{20}C_2) + 3^2({}^{20}C_3) + \dots\dots + 20^2\,{}^{20}C_{20}$ $= A \times 2^{\beta}$, then the ordered pair (A, β) is equal to.

Sol^n – $0^2 \cdot {}^{20}C_0 + 1^2 \cdot {}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \dots\dots + 20^2 \cdot {}^{20}C_{20}$

$$\sum_{r=0}^{20} r^2 \cdot {}^{20}C_r = ?$$

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 \cdot x^1 + {}^{20}C_2 \cdot x^2 + \dots\dots + 20 \cdot {}^{20}C_{20}\, x^{20}$$

$$20(1+x)^{19} = 0 + {}^{20}C_1 + 2 \cdot {}^{20}C_2 \cdot x + \dots\dots + 20 \cdot {}^{20}C_{20} \cdot x^{19}$$

$$20(1+x)^{19}x = 0\cdot x + {}^{20}C_1\cdot x + 2\cdot {}^{20}C_2\cdot x^2 + \cdots\cdots + 20\cdot {}^{20}C_{20}\cdot x^{20}$$

diff. w.r.t x.

$$20(1+x)^{19} + 380(1+x)^{18}\cdot x = 0 + 1^2\cdot {}^{20}C_1 + 2^2\cdot {}^{20}C_2 x + 3^2\cdot {}^{20}C_3 x^2 + \cdots\cdots + 20^2\cdot {}^{20}C_{20}\cdot x^n.$$

put $x = 1$

$$20\times 2^{19} + 380\times 2^{18} = 1^2\cdot {}^{20}C_1 + 2^2\cdot {}^{20}C_2 + \cdots\cdots + 20^2\cdot {}^{20}C_{20}.$$

$$= 2^{18}\cdot(420) = A\cdot 2^{\beta}$$

$$\therefore A = 420;\ \beta = 18.$$

Qus. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot {}^{50-r}C_{25-r}\right\} = K\cdot\left\{{}^{50}C_{25}\right\}$, then K is equal to

a) 2^{25}

b) 2^{24}

c) $(25)^2$

d) $2^{25}-1$

Solⁿ

$${}^{50}C_0\cdot {}^{50}C_{25} + {}^{50}C_1\cdot {}^{49}C_{24} + {}^{50}C_2\cdot {}^{48}C_{23} + \cdots\cdots + {}^{50}C_{25}\cdot {}^{25}C_0$$

Coefficient of x^{25} in $\Big({}^{50}C_0(1+x)^{50} + {}^{50}C_1(1+x)^{49} + {}^{50}C_2(1+x)^{48} + \cdots + {}^{50}C_{25}(1+x)^{25} + \underline{{}^{50}C_{26}(1+x)^{24} + \cdots + {}^{50}C_{50}(1+x)^0}\Big)$

Coefficient of x^{25} in $(1+x+1)^{50}$

" " x^{25} in $(2+x)^{50}$

$$= {}^{50}C_{25}\cdot 2^{50-25}$$

$$= \underbrace{2^{25}}_{K}\cdot {}^{50}C_{25}$$

Qus. Let $(x+10)^{50} + (x-10)^{50} = a_0 + a_1 x + a_2 x^2 + \cdots\cdots + a_{50}x^{50}$, for all $x\in R$; then $\frac{a_2}{a_0}$ is equal to.

Solⁿ $\Rightarrow (x+10)^{50} + (x-10)^{50} = a_0 + a_1 x + a_2 x^2 + \cdots + a_{50}x^{50}.$

$$\sum_{r=0}^{50} {}^{50}C_r\cdot x^{50-r}\cdot 10^r + \sum_{r=0}^{50} {}^{50}C_r\cdot x^{50-r}\cdot 10^r\cdot(-1)^r.$$

$a_2/a_0 = ?$

$a_2 = \text{Coeff. of } x^2 = {}^{50}C_{48} \cdot 10^{48} + {}^{50}C_{48} \cdot 10^{48}$
$= 2 \times {}^{50}C_{48} \times 10^{48}.$

$a_0 = \text{Coeff. of } x^0 = {}^{50}C_{50} \cdot 10^{50} + {}^{50}C_{50} \cdot 10^{50}$
$= 2 \times {}^{50}C_{50} \times 10^{50}.$

$$\therefore a_2/a_0 = \frac{2 \times {}^{50}C_{48} \times 10^{48}}{2 \times {}^{50}C_{50} \times 10^{50}} = \frac{\cancel{50} \times 49}{2 \times 1 \times \cancel{10^2}} = 49/4 = 12.25.$$

Ques. The total number of 3-digit numbers, whose sum of digit is 10 is 54.

Sol^n $a+b+c = 10$

$0 \le b, c \le 9$; $1 \le a \le 9$

$0 \le a-1 \le 8.$

$a' = a-1.$

$a' = a \longrightarrow a = 10$ $(10, 0, 0)$ ✗.

$(a-1+b+c) = 9.$

$(a'+b+c) = 9$

$0 \le a', b, c \le 9$

$\left.\right\}$ ${}^{9+3-1}C_{3-1}$
$= {}^{11}C_2 = 55$

$(10, 0, 0)$ is included in sol^n

$\Rightarrow 55 - 1 = 54.$

Use of Binomial and Multinomial theorems in permutation and combination problems

We knows.

$x_1 + x_2 + \dots + x_r = n$, $x_1, x_2, x_3, \dots, x_r \in W$

$0 \le x_1, x_2, \dots, x_r \le n$ $n \in W$

No. of solution $= {}^{n+r-1}C_{r-1}$

No. of terms in Multinomial

$$(x_1+x_2+\dots+x_r)^n = \sum \frac{n!}{P_1! \cdot P_2! \dots P_r!} x_1^{P_1} \cdot x_2^{P_2} \dots$$

Setup of generating function

$$= \underbrace{(x^0 + x^1 + x^2 + \dots + x^n)}_{\text{for } x_1} \cdot \underbrace{(x^0 + x^1 + x^2 + \dots + x^n)}_{\text{for } x_2} \dots \underbrace{(x^0 + x^1 + x^2 + \dots + x^n)}_{\text{for } x_r}$$

Nr of non-negative solution

$= \text{Coefficient of } x^n \text{ in } (x^0 + x^1 + \dots + x^n)^r$

$= \text{Coeff. of } x^n \text{ in } \left(\frac{x^{n+1}-1}{x-1}\right)^r$

$= \text{Coeff. of } x^n \text{ in } (x^{n+1}-1)^r (x-1)^{-r}$

$= \text{Coeff. of } x^n \text{ in } (-1)^r (1-x)^{-r}$

No. of non-negative solⁿ $= {}^{n+r-1}C_{r-1}$.

Qus. $x+y+z=4$

$0 \le x, y, z \le 4$, $x, y, z \in W$

find (x, y, z)

Sol^n Method 1.

No. of triplet $(x, y, z) = {}^{4+3-1}C_{3-1} = {}^{6}C_{2} = 15$

Method 2.

(Using generating function)

$p \rightarrow$ some const.

$$f(p) = (p^0 + p^1 + p^2 + p^3)^3$$

$$= \left(\frac{1-p^4}{1-p}\right)^3$$

$$f(p) = (1-p^4)^3 \cdot (1-p)^{-3}$$

No. of non-negative Sol^n = Coeff. of p^4 in $(1-p^4)^3 (1-p)^{-3}$

= Coeff. of p^4 in

$$\left(1 + 3p + (-3)(-3-1) \cdot \frac{p^2}{2!} + \cdots\right)$$

$$= \frac{(-3)\times(-4)\times(-5)\times(-6)}{24} = 15$$

NOTE:—

Coefficient of x^n in $(1-x)^{-r} = {}^{n+r-1}C_{r-1}$

or, ${}^{n+r-1}C_{n}$

Qus. In an examination, the maximum marks for each of three papers are 50. Maximum marks for the fourth paper are 100. find the number of ways in which the candidate can score 60% marks in aggregate.

Sol^n –

Papers	M. marks	Obtained Marks
P_1	50	x_1
P_2	50	x_2
P_3	50	x_3
P_4	100	x_4

$$x_1 + x_2 + x_3 + x_4 = 60\% \text{ of } 250$$
$$x_1 + x_2 + x_3 + x_4 = 150$$
$$0 \le x_1, x_2, x_3 \le 50 \quad ; \quad 0 \le x_4 \le 100$$

No. of ways the candidate can score 60% marks.

= No. of ways of the ~~sh~~ selection set (x_1, x_2, x_3', x_4)

$$= \text{Coeff. of } x^{150} \text{ in } (x^0 + x^1 + x^2 + \dots + x^{50})^3 (x^0 + x^1 + x^2 + \dots + x^{100})$$

$$= \text{Coeff. of } x^{150} \left(\frac{1-x^{51}}{1-x}\right)^3 \left(\frac{1-x^{101}}{1-x}\right)^1$$

$$= \text{Coeff. of } x^{150} \text{ in } (1-x^{51})^3 (1-x^{101})(1-x)^{-4}$$

$$= \text{Coeff. of } x^{150} \text{ in } (1+3x^{102} - 3x^{51} - x^{153})(1-x^{101})(1-x)^{-4}.$$

$$= \text{Coeff. of } x^{150} \text{ in } (1 + 3x^{102} - 3x^{51} - x^{153} - x^{101} - 3x^{203} + 3x^{152} + x^{254})(1 + {}^4C_1 x^1 + {}^5C_2 x^2 + \dots + {}^{4+r-1}C_r x^r + \dots)$$

$$= {}^{153}C_{150} + 3 \cdot {}^{51}C_{48} - 3 \times {}^{102}C_{99} - {}^{52}C_{49}.$$

$$\Rightarrow (1-x)^{-n} = 1 + nx + (-n)(-n-1)\frac{x^2}{2!} - (-n)(-n-1)(-n-2)\frac{x^3}{3!} + \dots$$

$$= 1 + {}^nC_1 x^1 + \frac{n(n+1)(n-1)!}{(n-1)!\,2!} x^2 + \dots$$

$$= 1 + {}^nC_1 x^1 + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + \dots + {}^{n+r-1}C_r x^r + \dots$$

Ques. In how many ways can we get a sum of at most 17 by throwing six distinct dice? In how many ways can we get a sum greater than 17?

Sol^n — Let the no.s be

$$x_1, x_2, x_3, x_4, x_5, x_6$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 17$$

$$1 \le x_1, x_2, x_3, \dots x_6 \le 6$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \in I^+$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + \underbrace{x_7}_{\text{Dummy Variable}} = 17$$

$$x_7 \le 0$$

$$0 \le x_7 \le 11$$

No. of ways = Coeff. of x^{17} in $(x^1 + x^2 + \dots + x^6)^6 (x^0 + x^1 + x^2 + \dots + x^{11})$

$= \text{Coeff. of } x^{17} \text{ in } x^6 \left(\frac{1-x^6}{1-x}\right)^6 \left(\frac{1-x^{12}}{1-x}\right)$

$= \text{Coeff. of } x^{17} \text{ in } x^6 (1-x^6)^6 (1-x^{12})(1-x)^{-7}$

$= \text{Coeff. of } x^{11} \text{ in } (1-x^6)^6 (1-x^{12})(1-x)^{-7}$

$= \text{Coeff. of } x^{11} \text{ in } (1-x^6)^6 (1-x)^{-7}$

$= \text{Coeff. of } x^{11} \text{ in } (1-6x^6)(1-x)^{-7}$

$= {}^{7+11-1}C_{11} - 6 \times {}^{7+5-1}C_5$

$= {}^{17}C_{11} - 6 \cdot {}^{11}C_5$

No. of ways we can get sum greater than 17

$= 6^6 - \{{}^{17}C_{11} - 6 \cdot {}^{11}C_5\}$

Derangement

Case 1.

Arranged $\boxed{E_1}$ $\boxed{E_2}$

L_1 L_2

Dearrangement

$\boxed{E_1}$ $\boxed{E_2}$

$\uparrow$ $\uparrow$

L_2 L_1 $\longrightarrow$ 1 ways

Case 2.

Arranged. $\boxed{E_1}$ $\boxed{E_2}$ $\boxed{E_3}$

$\uparrow$ $\uparrow$ $\uparrow$

L_1 L_2 L_3

Dearrangement

$\left.\begin{matrix} L_3 & L_1 & L_2 \\ L_2 & L_3 & L_1 \end{matrix}\right\} \longrightarrow$ 2 ways.

There are 'n' letters and 'n' corresponding envelopes. The no. of ways that all the letters can be placed in the envelopes (one letter in each envelopes) so that no letter is pla in correct envelopes is given by:

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \dots\dots + \frac{(-1)^n}{n!}\right]$$

$\longrightarrow D_1 = 0$

$\longrightarrow D_2 = 1$

$\longrightarrow D_3 = 2$

$\longrightarrow D_4 = 9$

$\longrightarrow D_5 = 44$.

Ex:– find the no. of ways that all letters of the word SWORD can be arranged such that no letter is in its original position.

→ No. of ways $= 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = 44$

Ex:– Let $f: A \rightarrow A$ be an invertible function where $A = \{1, 2, 3, 4, 5, 6\}$. find the number of the ~~functio~~ functions in which at least three elements have self Image.

→ Required no. of f^n $= {}^6C_3 \times 3! \times \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)$

$+ {}^6C_4 \times 2! \left(1 - \frac{1}{1!} + \frac{1}{2!}\right) + {}^6C_5 \times 1! \times \left(1 - \frac{1}{1!}\right) + {}^6C_6$

$= 40 + 15 + 0 + 1$

$= 56$

State Level Exercise

20. Number of ways of selectioning of 8 letters from 24 letters of which 8 are a, 8 are b and the rest are unlike, is given by

1. 2^7 2. $8 \cdot 2^8$ 3. $10 \cdot 2^7$ 4. None of these.

Sol^n –

24 Letters → Type-a (8 letters), Type-b (8 letters), Different (8 letters) $(x_3, x_4, \dots, x_{10})$ ⟹ Combination of 8 letters.

$$\underbrace{x_1}_{a} + \underbrace{x_2}_{b} + \underbrace{x_3 + x_4 + \dots\dots + x_{10}}_{\text{Different}} = 8$$

$0 \le x_1 \le 8$ Type-a letter

$0 \le x_2 \le 8$ Type-b letter

$$\left.\begin{array}{l} x_3 = 0, 1 \\ x_4 = 0, 1 \\ \vdots \\ x_{10} = 0, 1 \end{array}\right] \quad x_i = 0 \text{ or } 1, \quad i = \{3, \dots, 10\}.$$

No. of ways of selecting 8 letters

$$= \text{Coeff. of } x^8 \text{ in } \underbrace{(x^0 + x^1 + \dots + x^8)}_{\text{for } x_1} \times \underbrace{(x^0 + x^1 + \dots + x^8)}_{\text{for } x_2} \times \underbrace{(x^0 + x^1)}_{\text{for } x_3} \times \dots \times \underbrace{(x^0 + x^1)}_{\text{for } x_{10}}$$

$$= \text{Coeff. of } x^8 \text{ in } \left(\frac{1-x^9}{1-x}\right)^2 (1+x)^8$$

$$= \text{coeff. of } x^8 \text{ in } (1 + \cancel{x^{18}} - \cancel{2x^9})(1-x)^{-2}(1+x)^8$$

$= \text{Coeff. of } x^8 \text{ in } (1-x)^{-2}(1+x)^8$

$= {}^8C_0 \cdot {}^9C_8 + {}^8C_1 \cdot {}^8C_7 + {}^8C_2 \cdot {}^7C_6 + {}^8C_3 \cdot {}^6C_5$

$+ {}^8C_4 \cdot {}^5C_4 + {}^8C_5 \cdot {}^4C_3 + {}^8C_6 \cdot {}^3C_2 +$

${}^8C_7 \cdot {}^2C_1 + {}^8C_8 \cdot {}^1C_0$

$= 9 + 64 + 196 + 336 + 350 + 224 + 84 + 16 + 1$

$= 10 + 260 + 350 + 560 + 100$

$= 110 + 610 + 560$

$= 720 + 560 = 1280 = 2^7 \cdot 10$

(c) → Ans